

Structural Analysis of Thin Tile Vaults and Domes: The Inner Oval Dome of the *Basílica de la Virgen de los Desamparados* in Valencia

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The *Basílica de la Virgen de los Desamparados*, built 1652-1667, is the most important building of the 17th Century in Valencia (Bérchez 1995). Its main feature is the big oval dome, which appears in the plan inscribed in a trapezoid, following a type proposed by Vignola in the second half of the 16th century [Sant Andrea in Via Flaminia, 1550; Sant-Anna dei Palafraneris 1572; see Lotz 1955]. This type was afterwards popular in late Renaissance and Baroque architecture of the 16th and 17th centuries throughout Europe. In Spain, several oval domes were built around 1600 (Rodríguez 1983), the biggest is that of the convento de Las Recoletas Bernardas in Alcalá de Henares, 1617-1626, with axis 25×18 m., and a height of 11m. (Schubert 1908). Thirty three years after the completion of the building, around 1700, a new dome was built, nested inside the original dome, to provide the support for an enormous fresco in honour of the Virgin. This inner dome is exceptionally slender and presents some unique characteristics that make it of exceptional interest for the history of masonry dome construction.

The construction and structure of the *Basílica de los Desamparados* was exhaustively studied during the preliminary studies made in the 1990s, before the works of restoration under the direction of the architect Ignacio Bosch. Some studies were published in 1999 (Roig and Bosch 1999); all the information concerning the studies and the subsequent restoration was compiled and published in a book (Bosch 2006). Thus, we have in this case an enormous amount of information about the construction and structure of the building. We are concerned here with the construction and struc-

tural behaviour of the innermost of the two nesting domes which form the cupola.

The outer dome

The original building had a simple dome when it was completed in 1667. It is an oval pointed dome, crowned by a lantern (Fig. 1). The dome has an oval plan of 19 by 15m. and a height of 12.70m. [$2/3$ of the main axis]. The shell of the dome is 310mm [a brick length, of bricks $310 \times 140 \times 40$ mm] and has eight ribs converging in an oval at the base of the lantern. However, these ribs project only half a brick or 150mm inside, being covered by plastered decoration. The ribs have variable breadth, from 900mm at the base to 450mm at the oval in the crown. At the base, the oval shows a proportion among the two axes of nearly $5/4$ and, probably, the overall geometry of its surface is obtained drawing pointed arches [for each vertical plane passing through the vertical axis of the dome] with vertical tangent and passing, either by the crown or by the oculus. The radii of the two principal [meridian] sections for the long and short axis are 12.5m. and 15m., respectively. The quotient radius/thickness is, then, 40-50, a figure similar to that of the dome of St. Paul's in London (Heyman 2010).

The inner dome

Around 1700 another inner dome was built to provide a support for a big fresco painted by Antonio Palomino: a panegyric to the glory of the

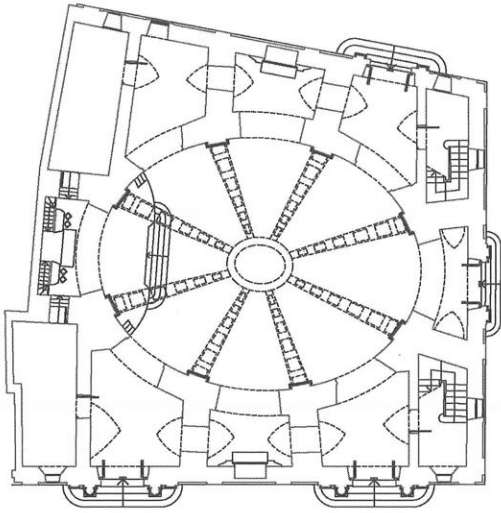


Fig. 1: Plan of the original dome of Los Desamparados (Bosch 2006).

Virgen de los Desamparados (Fig. 2). The fresco was finished in 1701. The building of the dome and the painting of the fresco took barely one year and a half. It was not uncommon in Spain in the 16th and 17th centuries to reform or modernize a church by building new tile vaults or domes under the original vaults. In Valencia, for example, under the gothic cross-vaults of the church of Santos Juanes a tile barrel vault with lunettes was built a decade earlier than the inner dome of Los Desamparados (Marín 2010) (Fig. 3).

The inner dome is a tile dome, built setting the tiles [thin bricks] "flat" with respect to the surface of the dome. It consists of two shells of tiles: the inner with tiles of 35mm thickness and the outer, 25mm thick; in the outer shell the tiles are set "breaking joints" with the inner shell. The mortar used was made of gypsum, which sets very quickly, so that the construction could proceed without centring. The intrados was plastered to receive the fresco with a 20mm thick plaster. Thus, the total thickness is 80mm. As we shall see the lower part of the dome runs parallel very near to the intrados of the outer dome, and for the above mentioned radii, the quotient radius/thickness is 150-180. The surface amounts to 530m.² and the dome weighs around 80 tons or 800 kN.

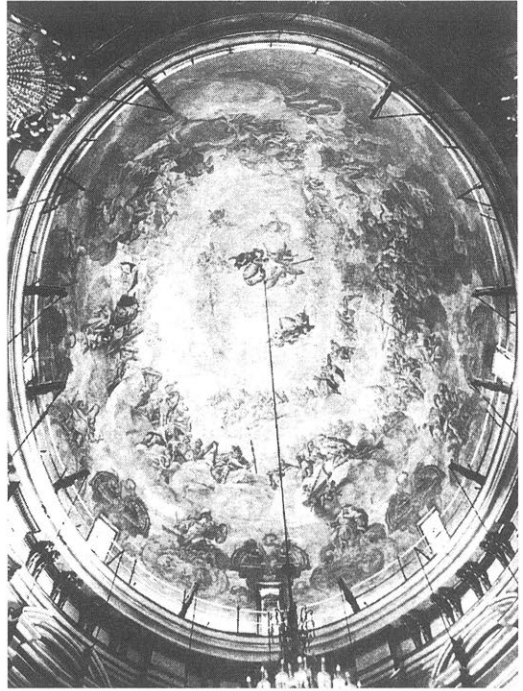


Fig. 2: Fresco of Antonio Palomino in the Basílica de los Desamparados (Bérchez 1995).

The dome springs from the big impost at the base of the original dome. Then, the shell was built concentric with the intrados and abutting against the brick projecting ribs of the outer dome. Before the construction the plastered decoration was removed to guarantee a uniform contact and to help to support the building of the first half of the dome by adhesion. Thus, the space between the two shells is at the springing of little more than half a brick, or ca. 160mm. Then, at an approximately half the height of the dome the shell departs from the form of the outer dome, forming an irregular ovoid. The two shells are now in contact by means of 126 wrought iron bars [15 × 15mm section], *encastrés* in the outer dome and terminated in "T" form to permit an anchorage on the extrados of the inner dome by means of two lateral tiles united to the anchorage and the extrados by gypsum mortar. The iron bars can be seen from the extrados of the inner dome [it is possible to enter through the hole at

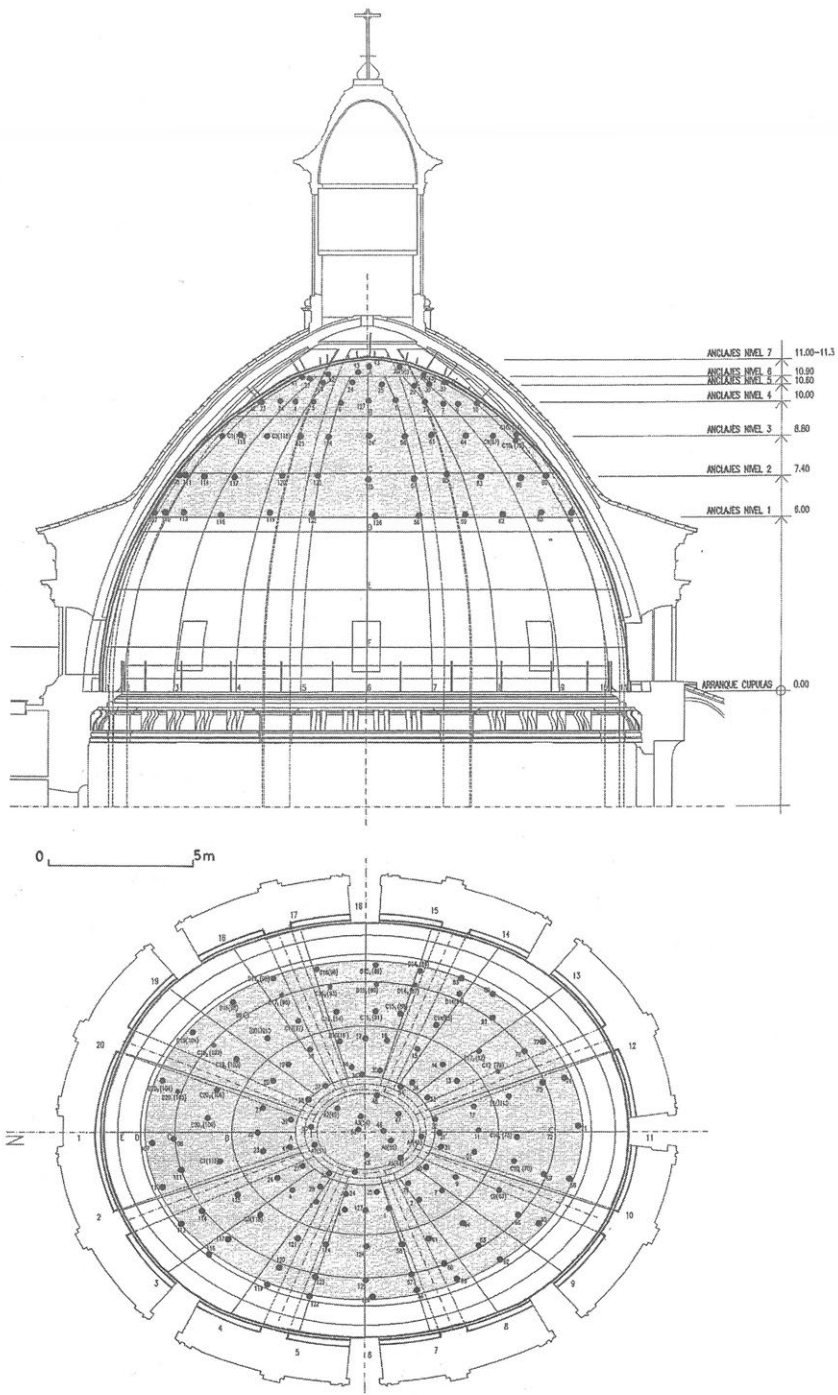
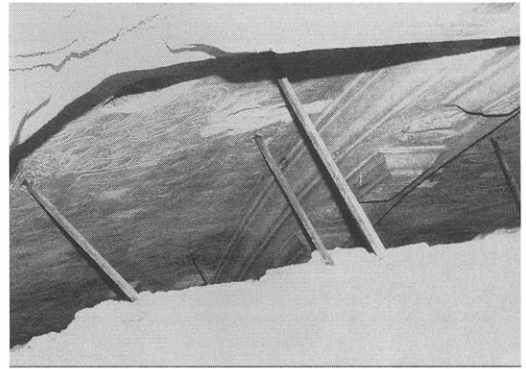


Fig. 3: Section and plan of the double oval dome of Los Desamparados (Bosch 2006).

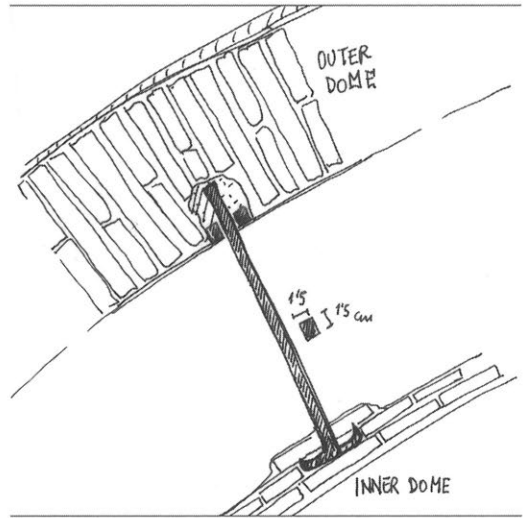
the crown and stand inclined and see them] but its position was determined using a metal detector. Their disposition is fairly regular: they are disposed in seven parallel rings spaced around 1.50-1.60m. along the meridian curve. This disposition of iron bars connecting two masonry domes is, as far as we know, unique, and was discovered by Bosch's team during the preliminary studies cited above (Bosch 1999). The disposition gave rise immediately to the question of their function. The bars were interpreted as "suspenders" hanging the inner dome from the outer dome. The inner dome was, then, a "false" dome, a ceiling to support the fresco (Fig. 4).¹ We will return to this fundamental aspect later.

A photogrammetric survey was made, locating 900 points of the intrados (Fig. 5). This permitted to draw with precision the two main sections of the inner dome, and a simplified form was assumed for later analysis. However, it appears that no study was made of the actual surface which shows visible irregularities. Such a study was made by the present author in writing an expertise on the safety of the inner dome (Fig. 6).² It is obvious that the lower part the inner dome runs parallel to the external dome, but, when the surface departs, and the iron bars appear, the surface shape is not regular, taking the form of a distorted ovoid, as can be appreciated in the perspective of Figure 6. The surface presents even zones with negative Gaussian curvature, which are perceived like bulges or dents. The sight may be alarming for a modern architect or engineer, but these irregularities are completely normal in tile domes built without centring. However, negative Gaussian curvature always provides an "arch" action and is, therefore, irrelevant. The author has observed them inspecting the extrados of several tile domes and vaults [see, for example, the study of the oval dome of the church of La Peregrina in Sahagún, Huerta 2010; Huerta and Fuentes 2010].

We are faced here with two kinds of questions. The first refer to the techniques of building of thin tile vaults and domes in Spain c. 1700. The detailed study made by Bosch's team describes exhaustively the physical constitution of the nested domes of Los Desamparados, gathering the bare facts, that is, the overall geometry, the mate-



(a)



(b)

Fig. 4: Iron bars connecting the two domes: a) Photograph (Bosch 1994); b) Sketch of the connections of the bar (author's drawing).

rials employed, etc. The second enters the field of interpretation. Why was the inner dome built in such a way? What is the function of the iron bars? Is the 80 ton dome hanging from them? What were the methods employed to control the shape of the dome during construction? And, eventually, is the inner dome safe? Yes, it has stood for 300 years, but is there a danger of collapse? Are the irregularities and distortions of the dome original or the result of gross deformations occurred along the centuries, which have led the dome to the verge of collapse?

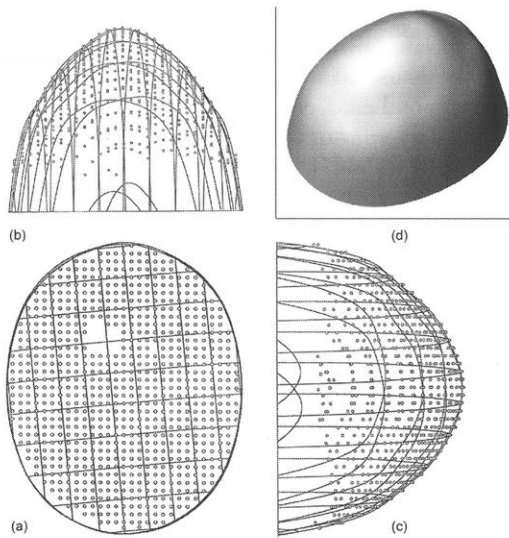


Fig. 5: Surface interpolated through the points of the photogrammetric survey: a-c) Plan and two main sections; d) Perspective.

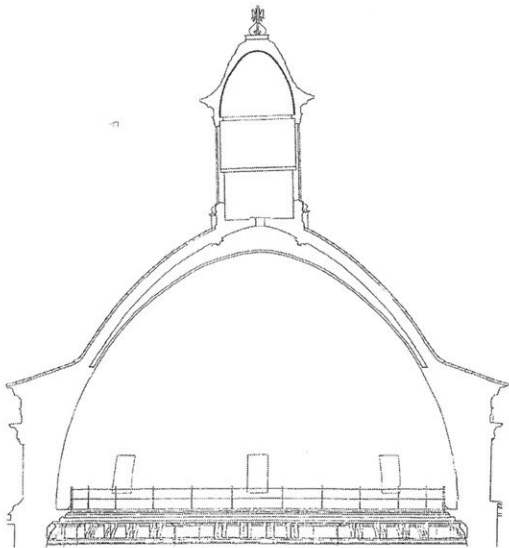


Fig. 6: Section drawn showing the buttressing effect of the ribs and drum of the outer dome (author's drawing after Bosch).

To answer the second kind of questions we need two frames of knowledge: the first refers to the field of construction history, that is, a

knowledge of the way tile vaults and domes were designed and built, safely, during the 16th and 17th century in Spain. This knowledge will allow us to try to think as the architect or mason around 1700 would have thought, to use the "logic" of construction to interpret the bare facts. The second refers to the question of structural behaviour of masonry vaults and domes and, in general, of masonry structures. We need a theoretical frame adequate to historic masonry, which considers the buildings such as they were built [piling bricks and stones, bonded with weak mortar, so that they form a stable structure], and not as we perhaps we would like them to be, made of steel or reinforced concrete so that we could apply the usual tools and modern codes of practice.

Tile vault and dome construction in 17th century Spain

Tile vaulting has been practiced in Spain for centuries. The oldest vaults are dated 1382 in Valencia, but Almagro (2001) has found evidence of tile vaults from the 12th century in Andalucía. It is, besides, a Mediterranean technique, and examples can be found also in Italy, France and the north of Africa. In Spain, since the 16th century were widely used because of the economy in the construction of parish churches and convents. For rich buildings, stone was considered a more noble material.

For the practice of design and building of tile vaults we have an invaluable source of information: the treatise written by Fray Lorenzo de San Nicolás, *Arte y uso de arquitectura*, published in Madrid in 1639. It is an extraordinary book [for Kubler (1957, 80) is "the best treatise ever written on the practice of architecture"]. The chapters on vault construction are particularly interesting, because this overall important part of building is usually barely explained in the architectural treatises. Fray Lorenzo describes the construction of the main types of vaults [barrel, cross and cloister vaults, domes, pendentives, lunettes, and stairs], all of them with three materials: tile vaulting, brick vaulting and stone.

Tile domes can be constructed without centring, using only a rod or strut, or maybe a light

formwork, to define the main geometry. Then the mason proceeds free-hand. As the building proceeds the vault must be backed to abut against the buttress system. The general rule is: the first third of the height should be backed with solid masonry and the second third by transverse [radial in the case of the dome] walls. The recommendation is first stated as a general rule, but Fray Lorenzo repeats it again for every kind of vault (Huerta 2003). In a typical dome over a square crossing surrounded by walls forming a cube and covered by a wooden roof, we find, invariably, solid filling the first third and then eight radial walls until the second third of the height: four following the diagonals and another four for the two main axes.

The theory of masonry structures

Masonry architecture is essentially different from modern architecture, and the structural analysis of historic buildings should take into account this fundamental difference. The usual theory of structures was developed during the 19th century for bar structures [either frames or trusses] made of steel or reinforced concrete. This theory is completely useless to understand the behaviour of even the simplest masonry structure, the arch. In the 18th and 19th centuries the theory of the masonry arch and vault was developed, taking into account the fundamental no-tension character of the masonry. Different methods of equilibrium and collapse analysis were devised, and they were used successfully (Heyman 1982). At the end of the 19th century, the development of the classical theory of elasticity swept away all this knowledge, and architects and engineers tried, unsuccessfully, to model masonry as an elastic isotropic material, notwithstanding the evidence that masonry is just the opposite, a discontinuous, heterogeneous, no-tension material, which besides is usually cracked.

In the 1960s Heyman demonstrated that masonry structures can be studied, rigorously, within the frame of Limit Analysis (Heyman 1995; 1996; 2008). The fundamental Safe Theorem of Limit Analysis justifies the use of only two of the three types of structural equations: equilibrium and material, making no statements about compatibility. The consequence can

be easily understood by any person with even a superficial knowledge on masonry building: If it is possible to find an equilibrium solution in compression of the masonry structure, then, this structure is safe will not collapse.

The 18th and 19th centuries arch and vault theories were, essentially, equilibrium analysis and are, therefore, basically correct. That an equilibrium solution in compression exists or not is a matter of geometry. It is the form of the building which guarantees its stability. The old building manuals contained, since gothic times, structural rules which define the size of the structural elements by means of geometrical rules [for example, the buttress of a barrel vault should be at least $1/3$ of the span, the thickness of the tambour of a Renaissance pointed dome should be $1/10$ of the diameter, etc. These geometrical rules were used for centuries or millennia (Heyman 1995; Huerta 2004)]. Then, we arrive, either through the exactness and rigour of modern structural theory, or by the experience of the best builders, to the same geometrical conclusions.

Thin masonry shells

The solution of equilibrium in compression must be contained within the masonry, that is, the internal forces in an arch, for example, can move freely [in equilibrium with the loads] in the space limited by the surfaces of intrados and extrados. In the arch the trajectory of forces can be easily imagined using Hooke's analogy with the hanging chain: the static of arches and cables is the same, and an arch functions like an inverted cable or hanging chain. If the arch has an adequate form and thickness, it is easy to imagine that the hanging chain supporting the loads can move within the masonry and there are, in fact, infinite equilibrium solutions with the masonry in compression.

In spatial structures there is more freedom as forces can be resolved in three directions. In particular in domes of revolution hoop, annular forces can develop and these forces allow to change the direction of the meridian forces. As a result, it is possible to find many different solutions of equilibrium. If we consider the hoop forces to be zero, then, the dome functions like a series of arches, "orange slices," converging at the crown. We may also impose other conditions, for example, that

all the internal forces are contained within the middle surface of the dome. This is the so-called membrane solution and, in this case, the dome need not have a finite thickness, can be infinitely thin, the limit being fixed by buckling, or by constructive limitations (Heyman 1977).

It is well known that in a thin dome or shell with vertical tangents at the springings, say, a hemisphere or an ellipsoid of revolution, the hoop forces are compressive in the upper part, but at a certain distance from the crown, tensile annular forces must appear to conduct the meridian forces within the middle surface. In a hemispherical dome of constant thickness tensions appear at 52 degrees from the crown, that is, nearly at $3/5$ or 0.6 [in fact the exact value is $\cos(51.8) = 0.62$] of the height of the dome. The masonry dome can be infinitely thin above this height, but below the thickness must increase so that the internal forces, which now follow the path of the hanging chain and diverge from the middle surface, could be contained within the masonry.

This is the function of the filling and the abutting walls used in practice: to provide a way of escape of the thrust of the upper part of the dome into the buttress system below. The height recommended by Fray Lorenzo is $2/3$ or 0.66 of the height, is larger than the above cited value of 0.62 and, therefore, safe. If the dome is pointed, the point of zero hoop stress descend, and it is also possible to build domes with inclined springings, which present no tension hoop stresses. This is the case of St. Paul's in London designed by Hooke and Wren with the form of a cubic parabola (Heyman 1998; 2010), but also of the dome in La Peregrina, where the master mason was able to produce an ovoid form working entirely in compression. The architect Rafael Guastavino Moreno designed also domes on inclined walls in the first half of the 20th century (Huerta 2003).

The inner dome of Los Desamparados: an interpretation

The master mason or architect who designed the inner dome of Los Desamparados was, in fact, following Fray Lorenzo's rules, which we have seen are entirely correct within the frame of modern

structural theory. The lower part of the dome was built concentric with the intrados of the outer dome but abutting against the projecting ribs of the outer dome [previously the decoration of plaster has been removed]. The ribs act as the solid fill and transverse walls prescribed by Fray Lorenzo. The short distance between the intrados and the inner shell permitted to control the form easily.

The construction continued in this way until a height of around five m., a little less than half the height of the inner dome [10.50m.]. Afterwards the shell should separate from the intrados to adopt the form of an ovoid dome, more adequate for the painting of the fresco. Two problems arose: first, how to control the shape of the shell; second, the inclination became less vertical and there was a danger of the shell collapsing inwards. It is true that a dome can have an oculus and be stable, but if the oculus does not have an adequate form, or present gross distortions, bending moments can appear which could not have been resisted by the very thin tile shell. This is the case in Los Desamparados where the upper horizontal sections present visible distortions. The decision taken shows enormous ingenuity. Iron rods of the same length were inserted *encastrés* in the masonry of the outer dome, at the same height and approximately the same inclination (Fig. 4). The heads of these bars defined a parallel section of the dome and helped the masons to build free-hand until this height. Besides, the heads of the iron bars with their T form were connected to the shell precluding any bending or instability.

As the domes separated, the lengths and inclination of the bars augmented. No doubt, their length was calculated before, probably from a drawing of the profile of the intrados of the outer dome drawn on the floor of the Basilica. Once the dome was closed, the iron bars had no function. A closed shell of double curvature is extraordinarily rigid.

The finished dome acts exactly in the same way as any tile dome, the upper part working in compression in very nearly a membrane state, and, abutting against the filling and transverse walls of the inferior part. In Los Desamparados this function is made by the eight projection ribs, helped also the windows lateral walls. The frightening section of figure 4, can be redrawn in figure 6,

cutting through the ribs. Now the buttressing action of the outer dome is evident, and the image is much more comforting.

The construction of the inner dome of the *Basílica de los Desamparados* in Valencia is a masterpiece of structural engineering. In a few months and with a minimum of material and auxiliary means, a tile dome of 530m.² of surface was built, with the modest objective of serving as a base for a fresco. The dome is not "hanging" from the iron bars; it is not a ceiling, a "false" dome. It is a

true thin tile shell designed and built with extreme ingenuity and economy. The upper part working in compression abuts against the ribs of the outer dome, which acts as the buttress walls of Fray Lorenzo. The iron bars lost their utility once the dome was closed, but are the hidden testimony of the constructive process, and should be preserved, now and in the future. The structure has demonstrated its safety for 300 years. This safety rests in an adequate form and a sufficient buttressing, a correct geometry which is the essence of masonry design.

NOTES

1. Bosch's team concluded that the inner dome was actually hanging from the outer dome. Considering that the iron bars were no longer adequate to fulfil this task, it was proposed to remove them and to place 292 aramid fibre rods to actually hang the inner dome (Bosch 2006). The present author, being asked by the Generalitat Valenciana for an expertise on the matter, expressed in a technical report (Huerta 2003) his complete disagreement with the diagnostic and the proposed intervention. The purpose of the present paper is not to reopen the debate, but to detail the construction and behaviour of one of the masterpieces of tile vault construction.

2. The photogrammetric survey was made in the 1990s. The digital file was kindly sent to the author by Ignacio Bosch in 2002. I would like to acknowledge the help of José Antonio García Ares and Paula Fuentes in handling the "cloud of points" with the Rhinoceros program. The program permits to interpolate a surface fixing some geometrical conditions. The main problem was that, as the measurements were taken from the floor, the impost produced a "shadow" and the position of the points of the base could not be measured. However, as the form of the oval of the outer dome was known and also the separation among the two shells, it was possible to guess the plan with sufficient approximation.

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